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Coupled plasmon–phonon modes in a two-dimensional electron gas in the presence of the Rashba effect

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Abstract

Elementary electronic excitation is studied theoretically for a two-dimensional electron gas in the presence of spin–orbit (SO) interaction induced by the Rashba effect. We find that in such a system, coupled plasmon–phonon excitation can be achieved via intra- and inter-SO electronic transitions. As a result, six branches of the coupled plasmon–phonon oscillations can be observed. The interesting features of these excitation modes are analysed.

Progress made in realizing spin-polarized electronic systems has led to recent proposals of novel electronic devices, such as spin transistors [1], spin waveguides [2], spin filters [3], and quantum computers [4]. One important aspect in the field of 'spintronics' is that of investigating electronic systems with finite spin splitting at zero magnetic field. It is known that in semiconductor-based two-dimensional electron gas (2DEG) systems, the zero-field spin splitting can be realized from an inhomogeneous surface electric field induced by the presence of the heterojunction. This feature is known as the Rashba effect [5]. In these systems the strength of the spin splitting and spin–orbit interaction (SOI) can be altered by applying a gate voltage [6] or varying sample growth parameters [7]. At present, most of the published work is focused on electronic and transport properties of 2DEGs in the presence of the SOI. In order to achieve a deeper understanding of these novel material systems and to explore further applications in practical devices, it is necessary for us to examine the roles which electronic many-body effects and phonons can play in a 2DEG with SO coupling.

Here we consider an interacting 2DEG where SOI and electron–phonon (e–p) interaction are present. Our aim is to obtain coupled plasmon–phonon excitation modes for this system. For a typical 2DEG in the xy-plane in narrow-gap semiconductors, such as InGaAs/InAlAs quantum wells, the noninteracting Schrödinger equation including the lowest order of SOI can be solved analytically [2]. Applying the electron wavefunctions to the electron–electron (e–e)

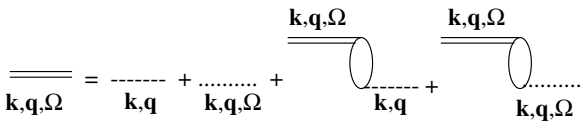

Figure 1. The effective e–e interaction (double solid lines) in the presence of phonon scattering. Here, the dashed line is the bare e–e interaction, the dotted line is induced by electron–phonon scattering, and the bubble refers is the bare pair bubble.

interaction Hamiltonian induced by the Coulomb potential, the space Fourier transform of the matrix element for bare e–e interaction is written as

$$V_{\alpha\beta}(\boldsymbol{k},\boldsymbol{q}) = V_q F_0(q) G_{\alpha\beta}(\boldsymbol{k},\boldsymbol{q}). \tag{1a}$$

Here, $\alpha = (\sigma',\sigma)$ with $\sigma = \pm 1$ referring to different SOIs, $\boldsymbol{k} = (k_x, k_y)$ is the electron wavevector along the 2D plane, $\boldsymbol{q} = (q_x, q_y)$ is the change of $\boldsymbol{k}$ during a scattering event, $V_q = 2\pi e^2/\epsilon_\infty q$ with ϵ_∞ being the high-frequency dielectric constant, and

$$G_{\alpha\beta}(\boldsymbol{k},\boldsymbol{q}) = \frac{1+\alpha A_{kq}}{2}\delta_{\alpha,\beta} + \mathrm{i}\frac{\alpha B_{kq}}{2}(1-\delta_{\alpha,\beta}) \tag{1b}$$

with $A_{kq} = (k + q\cos\theta)/|\boldsymbol{k}+\boldsymbol{q}|$, and $B_{kq} = q\sin\theta/|\boldsymbol{k}+\boldsymbol{q}|$, θ being the angle between $\boldsymbol{k}$ and $\boldsymbol{q}$. Furthermore, the space Fourier transform of the matrix element for bare e–p interaction can be written as

$$V_\alpha^{ph}(\boldsymbol{k},\boldsymbol{q};\Omega) = \sum_{q_z} D_0(\omega_Q,\Omega)|U_\alpha(\boldsymbol{k},\boldsymbol{Q})|^2, \tag{2}$$

where $\boldsymbol{Q} = (\boldsymbol{q}, q_z)$ is the phonon wavevector, ω_Q is the phonon frequency, $D_0(\omega_Q,\Omega) = 2\hbar\omega_Q/[(\hbar\Omega)^2 - (\hbar\omega_Q)^2]$ is the bare phonon propagator, $|U_\alpha(\boldsymbol{k},\boldsymbol{q})|^2 = |W_Q|^2 G_0(q_z) A_\alpha(\boldsymbol{k},\boldsymbol{q})$ is the square of the e–p interaction matrix element, $A_\alpha(\boldsymbol{k},\boldsymbol{q}) = (1+\alpha A_{kq})/2$ is a spin-dependent element, and W_Q is the e–p coupling coefficient. It should be noted that in contrast to the case for a conventional 2DEG (C2DEG) for which the bare e–e and bare e–p interactions do not depend on $\boldsymbol{k}$ [8], $V_{\alpha\beta}(\boldsymbol{k},\boldsymbol{q})$ and $V_\alpha^{ph}(\boldsymbol{k},\boldsymbol{q};\Omega)$ for a 2DEG with SOI depend not only on $\boldsymbol{q}$ but also on $\boldsymbol{k}$, because the spin splitting depends explicitly on $\boldsymbol{k}$. In the present study, we consider the case of a narrow quantum well in which only one electronic subband is present. Thus, $F_0(q) = \int \mathrm{d}z_1 \int \mathrm{d}z_2\, |\psi_0(z_1)|^2|\psi_0(z_2)|^2 \exp(-q|z_1 - z_2|)$ and $G_0(q_z) = |\langle 0|\mathrm{e}^{\mathrm{i}q_z z}|0\rangle|^2$, with $|0\rangle = \psi_0(z)$ being the electron wavefunction along the growth direction.

From the electron energy spectrum obtained by solving the Schrödinger equation, we derive the retarded and advanced Green functions for electrons. Using these Green functions, $V_{\alpha\beta}(\boldsymbol{k},\boldsymbol{q})$ and $V_\alpha^{ph}(\boldsymbol{k},\boldsymbol{q};\Omega)$, in a diagrammatic self-consistent theory [9] (also see figure 1), the effective e–e interaction is given by

$$V_{eff}(\boldsymbol{k},\boldsymbol{q};\Omega) = [V_{\alpha\beta}(\boldsymbol{k},\boldsymbol{q}) + V_\alpha^{ph}(\boldsymbol{k},\boldsymbol{q};\Omega)]\epsilon_{\alpha\beta}^{-1}(\boldsymbol{k},\boldsymbol{q};\Omega). \tag{3}$$

Here,

$$\epsilon_{\alpha\beta}(\boldsymbol{k},\boldsymbol{q};\Omega) = \delta_{\alpha,\beta}\delta(\boldsymbol{k}) - [V_{\alpha\beta}(\boldsymbol{k},\boldsymbol{q}) + V_\alpha^{ph}(\boldsymbol{k},\boldsymbol{q};\Omega)]\Pi_\beta(\boldsymbol{k},\boldsymbol{q};\Omega) \tag{4}$$

is the dielectric function matrix element and

$$\Pi_{\sigma'\sigma}(\boldsymbol{k},\boldsymbol{q};\Omega) = \frac{f[E_{\sigma'}(\boldsymbol{k}+\boldsymbol{q})] - f[E_\sigma(\boldsymbol{k})]}{\hbar\Omega + E_{\sigma'}(\boldsymbol{k}+\boldsymbol{q}) - E_\sigma(\boldsymbol{k}) + \mathrm{i}\delta} \tag{5}$$

is the pair bubble in the absence of e–e coupling with $f(E)$ being the Fermi–Dirac function. In equation (5),

$$E_\sigma(\boldsymbol{k}) = E_\sigma(k) = \hbar^2 k^2/2m^* + \sigma\alpha_R k \tag{6}$$

is the energy spectrum of a 2DEG in the presence of SOI, with m^* being the electron effective mass and α_R the Rashba parameter which measures the strength of the SOI.

For a 2DEG with SOI, the effective e–e interaction and dielectric function matrix depend not only on $\boldsymbol{q}$ but also on $\boldsymbol{k}$, in contrast to those for a C2DEG. After summing $\epsilon_{\alpha\beta}(\boldsymbol{k}, \boldsymbol{q}; \Omega)$ over $\boldsymbol{k}$ and setting $1 = (++)$, $2 = (+-)$, $3 = (-+)$, and $4 = (--)$, the dielectric function matrix for a 2DEG with Rashba spin splitting in the presence of e–p scattering is obtained as

$$\epsilon = \begin{pmatrix} 1 + a_1 + b_1 & 0 & 0 & a_4 \\ 0 & 1 + a_2 + b_2 & a_3 & 0 \\ 0 & a_2 & 1 + a_3 + b_3 & 0 \\ a_1 & 0 & 0 & 1 + a_4 + b_4 \end{pmatrix}. \tag{7}$$

In (7), $a_j = -[V_q F_0(q)/2] B_j(\boldsymbol{q}, \Omega)$ and $b_j = -\sum_{q_z} D_0(\omega_Q, \Omega)|W_Q|^2 G_0(q_z) B_j(\boldsymbol{q}, \Omega)$ are induced respectively by e–e and e–p interaction. $B_j(\boldsymbol{q}, \Omega) = \sum_{\boldsymbol{k}} (1 \pm A_{\boldsymbol{kq}}) \Pi_j(\boldsymbol{k}, \boldsymbol{q}; \Omega)$, where the upper (lower) case refers to $j = 1$ or 4 for intra-SO transitions ($j = 2$ or 3 for inter-SO transitions). The determinant of the dielectric function matrix is then given by

$$|\epsilon| = [(1 + a_1 + b_1)(1 + a_4 + b_4) - a_1 a_4][(1 + a_2 + b_2)(1 + a_3 + b_3) - a_2 a_3] \tag{8}$$

which results from intra- and inter-SO electronic transitions. Thus, the modes of coupled plasmon–phonon excitation are determined by $\mathrm{Re}|\epsilon| \to 0$.

In the present study, we consider an InGaAs-based 2DEG in which electrons interact strongly with longitudinal optical (LO) phonons through the Fröhlich coupling. For electron interaction with LO phonons, $\omega_Q \to \omega_{LO}$, the LO-phonon frequency at the long-wavelength limit, $|W_Q|^2 = 2\pi e^2 \hbar \omega_{LO} (\epsilon_\infty^{-1} - \epsilon_s^{-1})/Q^2$, with ϵ_s and ϵ_∞ being respectively the static and high-frequency dielectric constants. At the long-wavelength (i.e., $q \ll 1$) and low-temperature (i.e., $T \to 0$) limit, we have

$$\begin{aligned} \mathrm{Re}|\epsilon| \simeq & \left[1 - \frac{\omega_p^2}{\Omega^2} \frac{\Omega^2 - \omega_{TO}^2}{\Omega^2 - \omega_{LO}^2} \left(1 - \frac{\omega_- - \omega_+}{\omega_0/2}\right)\right] \\ & \times \left[1 - \frac{\omega_p^2}{\omega_0 \Omega} \frac{\Omega^2 - \omega_{TO}^2}{\Omega^2 - \omega_{LO}^2} \ln\left(\frac{\Omega + \omega_-}{\Omega - \omega_-} \frac{\Omega - \omega_+}{\Omega + \omega_+}\right)\right]. \end{aligned} \tag{9}$$

Here, the first (second) term on the right-hand side is induced by intra-SO (inter-SO) transitions, $\omega_{TO} = \sqrt{\epsilon_\infty/\epsilon_s}\,\omega_{LO}$ is the TO-phonon frequency, $\omega_\pm = 4\alpha_R \sqrt{\pi n_\pm}/\hbar$ with $n_\pm$ being the electron density in the $\pm$ spin channel, $\omega_0 = 16\pi n_e \hbar/m^*$, and $\omega_p = (2\pi e^2 n_e q/\epsilon_\infty m^*)^{1/2}$ is the plasmon frequency of a 2DEG in the absence of SOI with $n_e = n_+ + n_-$ being the total electron density of the system. Moreover, it can be shown that at the low-temperature limit, the electron density in different SOs is

$$n_\pm = (n_e/2) \mp (k_\alpha/2\pi)\sqrt{2\pi n_e - k_\alpha^2} \tag{10}$$

for the case of $n_e > k_\alpha^2/\pi$ with $k_\alpha = m^* \alpha_R/\hbar^2$. When $n_e \leqslant k_\alpha^2/\pi$, only spin-down states are occupied by electrons and, therefore, $n_+ = 0$ and $n_- = n_e$.

In the presence of SOI, the collective excitation from a 2DEG can be achieved via electron transitions in different spin channels. From equation (9), we see that the coupled plasmon–phonon frequency induced by intra-SO excitation is given by

$$\Omega_+ = \omega_{LO} + \frac{a}{2}\left(1 - \frac{\epsilon_\infty}{\epsilon_s}\right)\frac{\omega_p^2}{\omega_{LO}} \qquad \text{and} \qquad \Omega_- = \omega_p \sqrt{a \frac{\epsilon_\infty}{\epsilon_s}}, \tag{11}$$

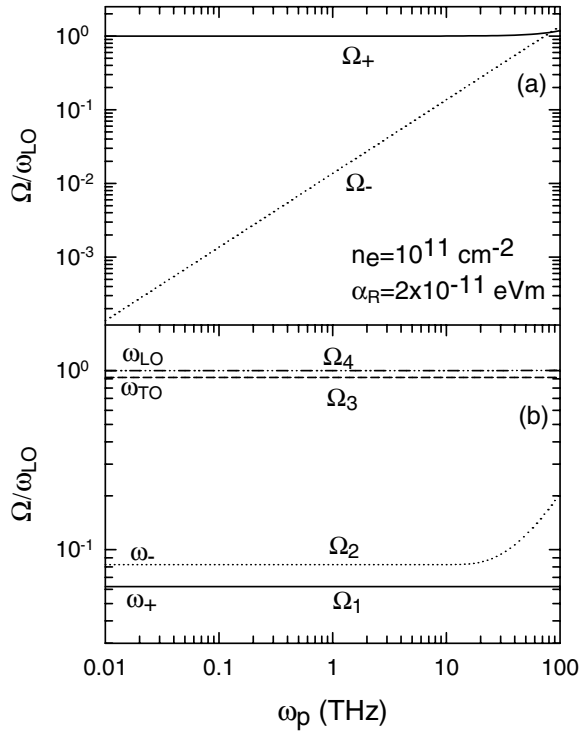

Figure 2. The dependence of the coupled plasmon–phonon frequency induced by intra-SO (in (a)) and inter-SO (in (b)) excitation of $\omega_p = (2\pi e^2 n_e q/\epsilon_\infty m^*)^{1/2}$ for fixed Rashba parameter α_R and total electron density n_e. Here, ω_{LO} and ω_{TO} are respectively the LO- and TO-phonon frequencies and $\omega_\pm = 4\alpha_R\sqrt{\pi n_\pm}/\hbar$.

where $a = 1 - 2(\omega_- - \omega_+)/\omega_0$; those induced by inter-SO transitions can be obtained by solving

$$\ln\left(\frac{\Omega+\omega_-}{\Omega-\omega_-}\frac{\Omega-\omega_+}{\Omega+\omega_+}\right) = \frac{\omega_0\Omega}{\omega_p^2}\frac{\Omega^2-\omega_{LO}^2}{\Omega^2-\omega_{TO}^2}. \tag{12}$$

Thus, two (four) branches of the coupled plasmon–phonon excitation can be observed via intra-SO (inter-SO) electronic transitions.

Now we present the results of our calculations for InGaAs-based quantum well structures. The material parameters are known [10]:

(1) the electron effective mass $m^* = 0.042\, m_e$ with m_e being the rest electron mass;
(2) the high-frequency and static dielectric constants are respectively $\epsilon_\infty = 12.3$ and $\epsilon_s = 14.6$; and
(3) the LO-phonon energy $\hbar\omega_{LO} = 30.9$ meV.

The dependences of the coupled plasmon–phonon frequencies induced by intra-SO (in (a)) and inter-SO (in (b)) excitation of $\omega_p = (2\pi e^2 n_e q/\epsilon_\infty)^{1/2}$ or q on the Rashba parameter α_R and on the total electron density n_e are shown respectively in figures 2–4. From these results and from equations (11) and (12), it can be found that at the long-wavelength limit, excitations with a frequency of about ω_{LO} can be generated via both intra-SO ($\Omega_+ \sim \omega_{LO}$) and inter-SO ($\Omega_4 \sim \omega_{LO}$) transitions and they depend very weakly on q and sample parameters (such as α_R

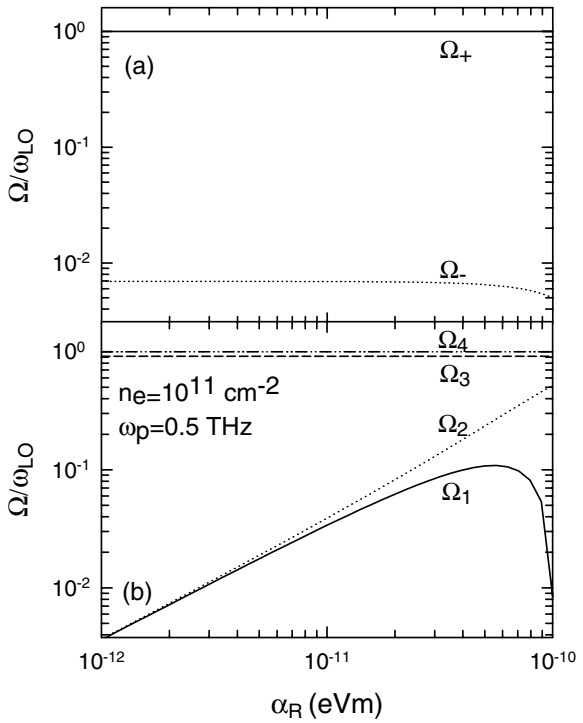

Figure 3. The coupled plasmon–phonon frequency induced by intra-SO (in (a)) and inter-SO (in (b)) transitions as a function of the Rashba parameter for fixed ω_p and n_e as indicated.

and n_e). Another mode induced by intra-SO transition, $\Omega_- \sim \omega_p \sim q^{1/2}$, is acoustic-like and depends weakly on α_R and n_e. The excitation with frequency of about $\Omega_3 \sim \omega_{TO}$ can only be generated via inter-SO transitions, and its dependences on q, α_R, and n_e are negligible, which implies that the TO-phonon mode can be excited via inter-SO transitions in an InGaAs-based 2DEG. Although Ω_1 and Ω_2 induced by inter-SO transitions should, in principle, depend on q via ω_p (see equation (12)), the numerical results shown in figure 2 suggest that at the long-wavelength limit, over a wide regime of ω_p or q, $\Omega_1 \to \omega_+ = 4\alpha_R\sqrt{\pi n_+}/\hbar$ and $\Omega_2 \to \omega_- = 4\alpha_R\sqrt{\pi n_-}/\hbar$ depend very little on q and can differ significantly from the phonon frequencies and from ω_p. This indicates that Ω_1 and Ω_2 are optic-like and they rely very strongly on sample parameters such as α_R and n_e (see figures 3 and 4).

The important conclusions drawn from this work are, for a 2DEG with SOI,

(1) LO-phonon excitation can be achieved via intra- and inter-SO electronic transitions;
(2) the TO-phonon mode can only be generated via inter-SO excitation; and
(3) Ω_1 and Ω_2 induced by inter-SO excitations are optic-like and very sensitive to sample parameters.

There have been many investigations of the coupled plasmon–phonon modes in semiconductor-based 2DEG systems carried out without including SOI [9, 11]. In comparing these results with those obtained in our present study (where SOI is present), we note some interesting features in the collective excitation modes, especially those excited through inter-SO transitions. It may be noted that state-of-the-art materials engineering and micro- and nano-fabrication techniques have made it possible to achieve an experimentally observable Rashba

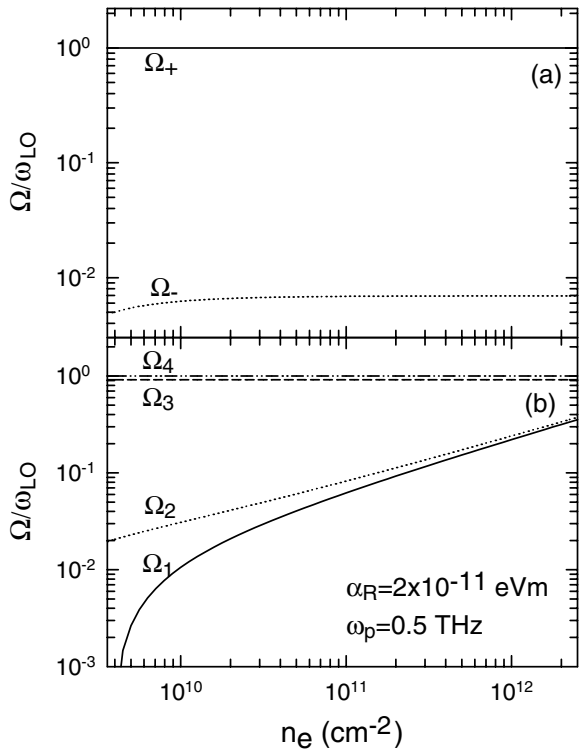

Figure 4. The coupled plasmon–phonon frequency caused by intra-SO (in (a)) and inter-SO (in (b)) excitations versus total electron density for fixed α_R and ω_p.

spin splitting in, e.g., InGaAs-based 2DEG systems. Very recent experimental results [12] for this system show that the Rashba parameter α_R can reach up to (3–4) $\times$ 10^{-11} eV m. Our results shown in figures 2–4 indicate that when $\alpha_R \geqslant 10^{-11}$ eV m, a significant separation between $\Omega_{1,2}$ induced by inter-SO transition can be achieved.

The present situation is that magneto-transport measurement is considered a powerful experimental technique and is the one most popularly used to identify Rashba spin splitting in a 2DEG [6, 7]. Using this technique to determine the Rashba parameter and electron density in different spin branches, one needs to apply high magnetic fields at low temperatures so that Shubnikov–de Haas oscillations are observable. From our investigations we suggest that the Rashba spin splitting can be probed by optical measurements. It should be noted that the dispersion of the coupled plasmon–phonon energies (or frequencies) is of the same order of magnitude as the energy splitting in different spin branches. In view of this, together with the fact that it is not so easy to measure the dispersion relation of the coupled plasmon–phonon modes in a 2DEG even without inclusion of SOI, we think that it may be difficult to measure optically the dispersion relation of the coupled plasmon–phonon mode in a 2DEG with SOI. In the absence of SOI, the dispersion of the coupled plasmon–phonon mode can only be determined by using techniques such as that of grating couplers [13]. However, Raman scattering [14] and ultrafast pump-and-probe experiments [15] have been carried out recently to study coupled plasmon–phonon modes in semiconductor-based 2DEG systems without SOI. These optical experiments conveniently measure optic-like excitation modes. Thus, we propose here that coupled plasmon–phonon modes in a spin-split 2DEG can be

detected optically because most of them are optic-like. In particular, if we can measure $\omega_{\pm} = 4\alpha_R\sqrt{\pi n_{\pm}}/\hbar$ (the magnitude of the frequencies and their separation are of the order of THz), we can determine the Rashba parameter and total electron density of the system. We see from figures 2–4 that when $\alpha_R \geqslant 10^{-11}$ eV m, $\Omega_2 - \Omega_1$ is much larger than $\Omega_4 - \Omega_3 \simeq \omega_{LO} - \omega_{TO}$. Because even ω_{LO} and ω_{TO} in InGaAs can be resolved in, e.g., Raman spectra [10], $\Omega_{1,2}$ induced by inter-SO transition can be more easily resolved in an optical experiment. Finally, we suggest that these theoretical predictions merit attempts at experimental verification.

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